SEEMAX INTERNATIONAL SCHOOL

SESSION-2024-25

Worksheet Name: Matrices **Standard: 12th Science** **Subject: Mathematics**

Q1. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations x + 3z = 9, –x + 2y – 2z = 4, 2x – 3y + 4z = –3.

Q2. Using matrices, solve the following system of equations:

4x + 3y + 2z = 60.

x + 2y + 3z = 45.

6x + 2y + 3z = 70.

Q3. if $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

1. $(A+B)' = A' + B'$
2. $(A - B)' = A' - B'$

Q4. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation A^2 - 3A - 7I = 0 and hence find A^{-1}.

Q5. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1} and hence solve the system of linear equations 2x – 3y + 5z = 11, 3x + 2y – 4z = – 5 and x + y – 2z = – 3.

Q6. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Q7. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A^T)^{-1}$.

Q8. Compute the adjoint of the following matrices:

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Verify that (adjoint A)A = |A|I = A (adjoint A) for the above matrices.

Q9. If $A = \begin{bmatrix} 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \end{bmatrix}$, then find a non-zero matrix C such that AC = BC.

Q10. Find the matrix A satisfying the matrix equation:

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Q11. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute (A + B) and (B - C).

Also, verify that A + (B - C) = (A + B) - C.

Q12. If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = A(A + I)$, where I is 3 × 3 unit matrix.

Q13. Find the matrix X satisfying the matrix equation.

$$X \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$$

Q14. If $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$, then find:

1. X + Y

2. 2X - 3Y

3. A matrix Z such that X + Y + Z is a zero matrix.

Q15. If X and Y are 2 × 2 matrices, then solve the following matrix equations for X and Y $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, 3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$.

Q16. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of ₹ 6,000. Three times the award money for Hard work added to that given for honesty amounts to ₹ 11,000. The award money given for Honesty and Hardwork together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

Q17. If A = $\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that I + A = (I - A) $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

Q18. For the matrices A and B, verify that (AB)' = B'A' where

1. $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$
2. $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Q19. Find the inverse of the following matrices:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Q20. If $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$ find A^{-1} and hence solve the system of equations x + 2y + 5z = 10, x – y – z = – 2 and 2x + 3y – z = – 11.

Q21. Find x, y, z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

Q22. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and prove that $A^2 - 4A - 5I = 0$.

Q23. Prove that $|A \text{ adj } A| = |A|^n$.

Q24. For the following pairs of matrices verify that $(AB)^{-1} = B^{-1} A^{-1}$:

$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

Q25. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, find A^{-1} and show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

Q26. Let $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. Find matrices X and Y such that X + Y = A, where X is a symmetric and Y is a skew-symmetric matrix.

Q27. Find the matrix X for which:
$$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$

Q28. If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

Q29. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify $(BA)^2 \neq B^2A^2$.

Q30. Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ sastisfies the equation $A^2 + 4A - 42I = 0$. Hence find A^{-1}.

Q31. If AB = BA for any two sqaure matrices, prove by mathematical induction that $(AB)^n = A^nB^n$.

Q32. If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that P(x).P(y) = P(x + y) = P(y)P(x).

Q33. Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$

Show that
$$[F(\alpha)G(\beta)]^{-1} = G(-\beta)F(-\alpha).$$

Q34. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

Q35. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$, find matrix X such that 2A + 3X = 5B.

Q36. If $A = [2 \quad 1]$, $B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then verify that $A(B + C) = (AB + AC)$.

Q37. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, Show that $A^2 - 5A + 7I = 0$. Hence, find A^{-1}.

Q38. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 - 3x - 7 = 0$. Thus, find A^{-1}.

Q39. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$ find x.

Q40. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, find a matrix C such that 3A + 5B + 2C is a null matrix.

Q41. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k such that $A^2 - 8A + kI = 0$.

Q42. If A = diag(a, b, c), show that $A^n = \text{diag}(a^n, b^n, c^n)$ for all positive integer n.

Q43. Express the matrix $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Q44. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then show that $(A + B)(A - B) \neq A^2 - B^2$.

Q45.

Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, compute $A^2 - B^2$.

Q46. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$, and I is the identity matrix of order 3, show that $A^3 = pI + qA + rA^2$.

Q47. Evaluate the following:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

Q48. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, verify $A(B + C) = AB + AC$.

Q49. Find the matrix A such that

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Q50. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$, compute $A^2 - 4A + 3I_3$.

Q51. Show that the matrix $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ is root of the equation $A^2 - 12A - I = 0$.

Q52. If w is a complex cube root of unity, show that.

$$\left(\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Q53. Solve the matrix equations:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

Q54. If $A = \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$, prove that

$A^2 = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2}\sin n\alpha \\ -\sqrt{2}\sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ for all $n \in N$.

Q55. Show that $AB \neq BA$ in the following cases:

$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$

Q56. If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that $P(x)P(y) = P(x + y) = P(y)P(x)$.

Q57. If $f(x) = x^3 + 4x^2 - x$, find f(A), where $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$

Q58. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^{-3} - 6A^2 + 5A + 11I_3 = 0$ Hence, find A^{-1}.

Q59. If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

Q60. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = x^2 - 2x - 3$, show that $f(A) = 0$

Q61. If $\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$, find x.

Q62. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k such that $A^2 = kA - 2I_2$.

Q63.

If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ find $A^2 - 5\,A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$

Q64. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find A^2 - 5A - 14I. Hence, obtain A^3.

Q65. Show that $A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ satisfies the equation x^2 - 12x + 1 = 0. Thus, find A^{-1}.

Q66. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $A^2 = 0$

Q67.
For the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$, show that A (adjoint A) = 0.

Q68. If $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, find A^2.

Q69. Construct $a_{2 \times 2}$ matrix, where,

1. $a_{ij} = \frac{(i-2j)^2}{2}$
2. $a_{ij} = |-2i + 3j|$

Q70.
If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, show that $AB = BA = O_{3\times3}$

Q71. Find matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

Q72. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find A^2 - 5A - 14I. Hence, obtain A^3.

Q73.
If $A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify $A^2 + A = A(A + I)$, where I is the identity matrix.

Q74.
Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Q75. Find a 2 × 2 matrix A such that.

$$A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6I_2$$

Q76. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find λ, μ so that $A^2 = \lambda A + \mu I$

Q77.
Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew symmetric.

Q78.
If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b and c.

Q79. Without using the concept of inverse of a matrix, find the matrix $\begin{bmatrix} x & y \\ z & u \end{bmatrix}$ such that $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

Q80.
If $A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that $A^2 = A^{-1}$.

Q81.
Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, Find A^T, B^T and verify that.

$(AB)^T = B^T + A^T$

Q82.
Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Compute $(AB)^{-1}$.

Q83. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -I$, then show that $(A + B)^2 = A^2 + B^2$.

Q84. Solve the matrix equations:

$$\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

Q85. For the matrices A and B, verify that $(AB)^T = B^T A^T$, where $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$

Q86. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value?

Q87. Express the following matrices as the sum of a symmentric and a skew symmentric matrix:

$$\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

Q88. Express the following matrices as the sum of a symmentric and a skew symmentric matrix:

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Q89.
If $f(x) = x^2 - 2x$, find f(A), where $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

Q90.
Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A (\text{adj } A) = |A| I_3$.

Q91. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$. Hence, find A^{-1}.

Q92.
If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find $A^T - B^T$.

Q93. Express the following matrices as the sum of a symmentric and a skew symmentric matrix:

$$\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

Q94. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, show that $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

Q95.
If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then prove that $A^2 - 4A - 5I = 0$.

Q96. Find the matrix X satisfying the equation:

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} X \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Q97.
If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 5 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and

And $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Q98. Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Q99. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, verify that $A^2 - 4A + I = 0$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $O \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Hence find A^{-1}.